SYMMETRY AND ANTIMATTER

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Hadi Vafaei, Reza Ershadinia

Cloud chamber photograph by C. D. Anderson of the first positron ever identified.
Image credit: en.wikipedia.org
Outline

1. Symmetries and Symmetry Groups
   (Masoud Mohammadi)

2. History of Antimatter
   (Reza Ershadinia)

3. Symmetry in Particle Physics
   (Sahand Seifnashri)

4. Symmetry in Condensed Matter Physics
   (Hadi Vafaei)
1. SYMMETRIES AND SYMMETRY GROUPS

- Introduction
- Symmetries and Conservation Laws
- Some Symmetries of Fundamental Laws of Physics
- Symmetry Group and Group Representation
What is symmetry?

Symmetry in everyday language refers to a sense of harmonious and beautiful proportion and balance.
Ancient people used to build their buildings symmetrically so as to look more beautiful.
What do we mean by saying symmetry in physics?

In physics, a symmetry of a physical system is a physical or mathematical feature of the system (observed or intrinsic) that is preserved or remains unchanged under some transformation.
There can be many transformations on a physical system. The set of these transformations together is called the group of the transformation. It is easy to show mathematically that this set has all the properties of a group.

\[ G = \{g_1, g_2, g_3, \ldots \} \]

For example the rotations about a special axis with different angles generate a group.
Now we can compare to systems or geometric shapes with each other. What do we mean when we say a system is more symmetric than the other?

When the number of the transformation group members, under which the system remains unchanged, is more than the other we say that the system is more symmetric than the other system.
Symmetries and Conservation Laws

Noether's theorem:

For each of the rules of symmetry there is a corresponding conservation law

Euler Lagrange Equations:

\[ \frac{\partial L}{\partial x} = \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{x}} \right) \quad \Rightarrow \]

\[ \frac{\partial L}{\partial x} = 0 \quad \Leftrightarrow \quad \frac{dP}{dt} = 0 \quad \left( P = \frac{\partial L}{\partial \dot{x}} \right) \]

(Translational Symmetry in x) \quad \Leftrightarrow \quad (Conservation of momentum)
Some Symmetries in Fundamental Laws of Physics

**Continues Symmetries:**

- Translation in space
- Translation in time
- Rotation
- Boost (Lorentz)
- Quantum-mechanical phase
- Gauge symmetry

**Discrete Symmetries:**

- Interchange of identical particles
- C (Charge conjugation)
- P (Parity)
- T (Time reversal)
- Supersymmetry
Symmetry Group and Group Representations

• For each system, the symmetries of that system form a symmetry group.

• A symmetry group is just the symmetries themselves and their product rules.

For a sphere, the symmetries can be abstracted as SO(3).

\[
\begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \theta & \sin \theta \\
0 & -\sin \theta & \cos \theta
\end{pmatrix}
\]
Symmetry Group and Group Representations

• Group representation of a symmetry group is a representation of the group elements by matrices.

• Since in quantum mechanics the states are in a Hilbert space we are interested in representing the group elements by unitary matrices.

For example the symmetries of sphere can be represented by $2 \times 2$ unitary matrices: $\sigma_x, \sigma_y, \sigma_z$
Why Group Representation Matters?

• Consider a quantum-mechanical system which is invariant under a set of operations (symmetries).

• Like electron which is invariant under rotations, so to identify how should the quantum states of the electron look like, we should study representations of rotation group into a Hilbert space.

• The electron lies in the $2$-dimentional representation of $SO(3)$: $\psi = \begin{pmatrix} a \\ b \end{pmatrix}$

• Furthermore, the $n$-dimentional representation of $SO(3)$, describes a system of spin $\frac{n-1}{2}$. 
2. A brief History of Antimatter
1905
Albert Einstein

- Special Relativity

\[ E = mc^2 \]
1912
Victor Franz Hess

- Found and studied cosmic rays.
- The positron and the muon were first discovered in cosmic rays.
- He shares Noble prize with Carl Anderson.
<table>
<thead>
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<th>Year</th>
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<td>Cosmic Ray</td>
</tr>
<tr>
<td>1926</td>
<td>Quantum Mechanics</td>
</tr>
<tr>
<td>1928</td>
<td>Relativistic QM</td>
</tr>
<tr>
<td>1932</td>
<td>Positron Discovery</td>
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</table>

1926-1930

**Erwin Schrödinger** *(top)*

**Werner Heisenberg** *(bottom)*

- Introduced QM to the world
- Their version of QM was not relativistic

\[
H(t)|\psi(t)\rangle = i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle
\]
1928
Paul Dirac
- Combined QM and SR
- Predicted that for every particle there is an antiparticle, exactly matching the particle with opposite charge.

\[ (i\gamma^\mu \partial_\mu - m)\psi^c = 0 \]
1932

Carl David Anderson

- As studying showers of cosmic particles saw a track left by "something positively charged, and with the same mass as an electron".
- He shares Noble prize with Victor Hess.
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1934

Ernest Lawrence

- Invented Cyclotron
- won the Nobel prize in physics, "for the invention and development of the cyclotron and for results obtained with it, especially with regard to artificial radioactive elements".

By Nobel foundation - here, Public Domain, here
- The name comes from Bev --billion electronvolt.
- It was designed to collide protons at the expected optimum energy for creating antiprotons.
1934 (Invention of Cyclotron)  
1954 (Bevatron)  
1955 (Hand-made Antiproton)  
1956 (Hand-made Antineutron)  
1964 (Surprising Kaon Decay)

1955
Owen Chamberlain
Emilio Segrè
Clyde Wiegand
Thomas Ypsilantis

- They made antiproton.
- Chamberlain and Segrè won Nobel prize for discovery of antiproton

From left, Emilio Segrè, Clyde Wiegand, Edward Lofgren, Owen Chamberlain, and Thomas Ypsilantis members of the team that discovered the antiproton
Photo: courtesy of Berkeley Lab
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<td>Bruce Cork, Glen Lambertson, Oreste Piccione, William Wenzel</td>
</tr>
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<td>1955</td>
<td>Hand-made Antiproton</td>
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From left: William Wenzel, Bruce Cork, Glen Lambertson, and Oreste Piccioni. Members of the team that discovered the antineutron at Bevatron. Photo: courtesy of Berkeley Lab.

They made antineutron.
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### 1964

James Cronin (top)
Val Fitch (bottom)

- Both won Nobel prize for “the discovery of violations of fundamental symmetry principles in the decay of neutral K-mesons"
1965
Antonino Zichichi (top)
Leon Lederman (bottom)

- Zichichi at CERN and Lederman at Brookhaven National Laboratory simultaneously observed the first antideuteron.
- Several hundred antiprotons of 2.1 GeV/c were produced and were kept circulating in ICE for a period of 85 hours ($3 \times 10^5$ s)
1981
Simon van der Meer (top)
Carlo Rubbia (bottom)

- They share Nobel prize for their decisive contributions to the large project, which led to the discovery of the field particles W and Z
1995
Walter Oelert

- This was the first time that antimatter particles had been brought together to make complete atoms, and the first step in a programme to make detailed measurements of antihydrogen.
The use of the **Antiproton Collector** holds the promise of delivering dense beams of $10^7$ protons per minutes and low energy (100 MeV/c)
3. SYMMETRY IN PARTICLE PHYSICS

- Gauge Symmetry
- Spontaneous Symmetry Breaking
- Discrete Symmetries of Particle Physics
Symmetries of Particle Physics

- **Space-time Symmetries:** Poincare Group
  - **Free Particle**

- **Gauge Symmetries:** $\text{SU}(3)_C \times \text{SU}(2)_L \times U(1)_H$
  - **Interactions**

- **Discrete Symmetries:** C, P, T
  - **Matter-Antimatter Symmetry**
Gauge Symmetry in QED

\[ \mathcal{L}_{\text{QED}} = \bar{\psi} (i \partial_\mu \gamma^\mu - m) \psi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - e \bar{\psi} \gamma^\mu \psi A_\mu \]

• The QED Lagrangian is invariant under the local \( U(1)_{\text{EM}} \) transformation:

\[
\begin{align*}
A_\mu & \rightarrow A_\mu + \partial_\mu \pi(x) \\
\psi & \rightarrow e^{-i\pi(x)} \psi
\end{align*}
\]

• The QED Lagrangian also invariant under global \( U(1)_{\text{EM}} \) gauge transformation,

\[ \psi \rightarrow e^{-i\alpha} \psi, \text{ which is associated with charge conservation.} \]
# Gauge Symmetries of the Standard Model (Before Higgs Mechanism)

<table>
<thead>
<tr>
<th>Gauge Group</th>
<th>Interaction</th>
<th>Force Carrier</th>
</tr>
</thead>
<tbody>
<tr>
<td>SU(3)\text{Color}</td>
<td>Strong</td>
<td>$g^1, g^2, \ldots, g^8$</td>
</tr>
<tr>
<td>SU(2)_{Left}</td>
<td>Weak</td>
<td>$W^1, W^2, W^3$</td>
</tr>
<tr>
<td>U(1)\text{Hypercharge}</td>
<td>Electroweak</td>
<td>$B$</td>
</tr>
</tbody>
</table>

Image credit: Fermilab
<table>
<thead>
<tr>
<th>Spin</th>
<th>Particle</th>
<th>Field</th>
<th>$SU(3)_C$ Representation</th>
<th>$SU(2)_L$ Representation</th>
<th>Hypercharge</th>
<th>Generations</th>
<th>Total Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spin $\frac{1}{2}$ (Fermion)</td>
<td>Quark</td>
<td>$Q = \begin{pmatrix} u \ d \end{pmatrix}_L$</td>
<td>3</td>
<td>2</td>
<td>1/6</td>
<td>3</td>
<td>$3 \times 2 \times 3 = 18$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$u_R$</td>
<td>3</td>
<td>1</td>
<td>2/3</td>
<td>3</td>
<td>$3 \times 1 \times 3 = 9$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$d_R$</td>
<td>3</td>
<td>1</td>
<td>-1/3</td>
<td>3</td>
<td>$3 \times 1 \times 3 = 9$</td>
</tr>
<tr>
<td></td>
<td>Lepton</td>
<td>$L = \begin{pmatrix} \nu \ e \end{pmatrix}_L$</td>
<td>1</td>
<td>2</td>
<td>-1/2</td>
<td>3</td>
<td>$1 \times 2 \times 3 = 6$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\nu_R$</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>$1 \times 1 \times 3 = 3$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$e_R$</td>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>3</td>
<td>$1 \times 1 \times 3 = 3$</td>
</tr>
<tr>
<td>Spin 1 (Gauge Boson)</td>
<td>Gluon</td>
<td>$g$</td>
<td>8</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>$8 \times 1 \times 1 = 8$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$W$</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>$1 \times 3 \times 1 = 3$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$B$</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>$1 \times 1 \times 1 = 1$</td>
</tr>
<tr>
<td>Spin 0 (Scalar)</td>
<td>Higgs</td>
<td>$H = \begin{pmatrix} H_u \ H_d \end{pmatrix}$</td>
<td>1</td>
<td>2</td>
<td>1/2</td>
<td>1</td>
<td>$1 \times 1 \times 1 = 2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Total: $61 + 1$</td>
</tr>
</tbody>
</table>
The Poincare group has two spin $\frac{1}{2}$ representations, which are known as Weyl spinors, the left-handed and right-handed Weyl spinors: $\psi_L$ and $\psi_R$.

For infinitesimal Lorentz transformations with rotation angles $\theta_j$ and boost angles $\beta_j$:

Left-handed Weyl spinors transform as: \[
\delta \psi_L = \frac{1}{2} (i \theta_j - \beta_j) \sigma_j \psi_L
\]

Right-handed Weyl spinors transform as: \[
\delta \psi_R = \frac{1}{2} (i \theta_j + \beta_j) \sigma_j \psi_R
\]

The Dirac fermions can be written as: \[
\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}
\]
and satisfy the Dirac equation:
\[
(i \partial_\mu \gamma^\mu - m) \psi = 0
\]
Electroweak Symmetry Breaking (Higgs Mechanism)

- In electroweak symmetry breaking: $SU(2)_L \times U(1)_H \rightarrow U(1)_{EM}$

- The Higgs field gets a non-zero vacuum expectation value, and breaks the electroweak symmetry.

- Then the Yukawa couplings of the Higgs field to other particles, give them mass:

$$\mathcal{L}_{Yuk} = -y \bar{L} H e_R + h. c. \quad \text{after SB}$$

$$\mathcal{L}_{Yuk} = -y H_0 (\bar{e}_L e_R + \bar{e}_R e_L) + \cdots = -y H_0 \bar{\psi} \psi$$

$$m_e = y H_0$$

Higgs Mexican Hat Potential:

Image Credit: www.quantum-bits.org
The Standard Model Before Symmetry Breaking

Spin 0 (Higgs Boson)

Hypercharge $\rightarrow Y$
Weak Isospin $\rightarrow T_3$
Gauge boson coupling

$H_0$

Electric Charge $Q = Y + T_3$

Spin 1/2 (Fermions)

Hypercharge (L) $\rightarrow Y$
Weak Isospin (L) $\rightarrow T_3$
Gauge boson coupling

Spin (GeV)

Generation

$Y$
$T_3$

Hypercharge (R) $\rightarrow Y$
Weak Isospin (R) $\rightarrow T_3$

Electric Charge $Q = Y + T_3$

Spin 1 (Gauge Bosons)

Fermion symbol

mass (GeV)

Fermion coupling

SU(3)$_{\text{COLOR}}$
SU(2)$_{\text{LEFT}}$
U(1)$_{\text{HYPERCHARGE}}$

246 GeV

Image credit: en.wikipedia.org
Discrete Symmetries: Parity (P) \( (\psi_L \leftrightarrow \psi_R) \)

- Classical Physics is invariant under parity, which means that one cannot distinguish between any physical process and its mirror image.

- In 1956 parity violation observed in a beta decay: \( ^{60}CO \rightarrow ^{60}Ni + e + \bar{\nu}_e \), which resulted in the 1957 Nobel Prize.

- In fact parity is explicitly broken in weak interactions, since only Left-handed fermions participate at weak interactions.

- But it is still a valid symmetry of the strong and electromagnetism interactions.
Charge Conjugation (C)

• QED and QCD are invariant under charge conjugation, which converts each particle with its antiparticle.

• But like parity, charge conjugation is also ‘maximally’ broken in weak interactions: A left-handed neutrino would be taken by charge conjugation into a left-handed antineutrino, which does not interact in the Standard Model.
CP

• It seems that CP is a symmetry of weak interactions, so a fundamental symmetry of nature.

• In 1964 CP violation was experimentally discovered in the decays of neutral kaons ($K^0: s\bar{d}$), which resulted in the 1980 Nobel Prize for its discoverers Cronin and Fitch.

• It took us almost ten years to explain this phenomenon in the context of the Standard Model of particle physics. In 1973, two Japanese physicists, Kobayashi and Maskawa, successfully explained CP violation through the complex phase of Cabibbo–Kobayashi–Maskawa matrix (CKM matrix).

In 2008, Kobayashi and Maskawa shared one half of the Nobel Prize in Physics "for the discovery of the origin of the broken symmetry which predicts the existence of at least three families of quarks in nature".
Time Reversal (T)

• Can one distinguish between a physical process and its time-reversed process?

• T-symmetry violation had not been observed experimentally until 2012, when the BaBar experiment at SLAC made first direct measurement of time-reversal violation.

• Theoretically, from the CPT theorem: CP violation $\Leftrightarrow$ T-symmetry violation
  Hence, T-Symmetry is also broken in weak interactions.
The Matter–Antimatter Asymmetry and CP Violation

• The big bang should have created equal amounts of matter and antimatter, with subsequent annihilation leaving neither behind. And yet, the observable universe consist entirely of matter (protons, neutrons, and electrons) with no antimatter.

• The weak force by itself can only explain a small amount of CP violation, not enough to leave matter for even a single galaxy. Some other hidden sources must have been responsible for the extra CP violation that led to the universe we observe.

• The Standard Model contains at least three sources of CP violation:
  - The CKM matrix in the quark sector
  - Electric dipole moment of the neutron in the strong sector
  - the PMNS matrix in the lepton sector (neutrino mixing angles)
    1. Driac CP phase
    2. Majorana CP phases
Symmetric Nobel Prizes

1. **The Nobel Prize in Physics 1957 - Chen Ning Yang and Tsung-Dao (T.D.) Lee**
   "for their penetrating investigation of the so-called parity laws which has led to important discoveries regarding the elementary particles"

2. **The Nobel Prize in Physics 1963 - Eugene Paul Wigner**
   "for his contributions to the theory of the atomic nucleus and the elementary particles, particularly through the discovery and application of fundamental symmetry principles"

   "for the discovery of violations of fundamental symmetry principles in the decay of neutral K-mesons"

4. **The Nobel Prize in Physics 2008 - Yoichiro Nambu**
   "for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics"
   **Makoto Kobayashi and Toshihide Maskawa:** "for the discovery of the origin of the broken symmetry which predicts the existence of at least three families of quarks in nature"

5. **The Nobel Prize in Physics 2013 - François Englert and Peter W. Higgs**
   "for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles, and which recently was confirmed through the discovery of the predicted fundamental particle, by the ATLAS and CMS experiments at CERN's Large Hadron Collider"
• It is a hypothetical symmetry that predicts each of the particles in the Standard Model has a partner with a spin that differs by $\frac{1}{2}$. So bosons are accompanied by fermions and vice versa.

• If it is true, it solves numerous open-problems of the Standard model, such as:

  - The hierarchy problem
  - Gauge coupling unification
  - Dark matter
  - String Theory

Supersymmetry remains an interesting, if mathematically elegant, hypothesis right now. It would explain why particles have the masses they do and shed light on dark matter. Many versions of the theory suggest the LHC’s current experiments will be energetic enough to produce the heavier supersymmetric particles — if they exist.

Image credit: www.discovermagazine.com
It is only slightly overstating the case to say that physics is the study of symmetry.

- Philip Warren Anderson (More is Different)
4. Symmetry in condensed matter physics

Outline:

• Symmetry and crystal structure
• Critical phenomena, phase transition and symmetry breaking
• Symmetries of the Hamiltonian
Symmetry & Crystal Structures
Crystals

- Atoms that are bound together, do so in a way that minimizes their energy.
- This most often leads to a periodic arrangement of the atoms in space.
- If the arrangement is purely periodic we say that it is crystalline.
Crystal = Lattice + Basis
Symmetry Operations

- Translational
- Reflection at a plane
- Rotation about an axis
  - Inversion through a point
- Glide (=reflection + translation)
- Screw (=rotation + translation)
Mirror Symmetry

$\sigma_d$

$\sigma_v$

$m$
Crystal Lattice

Bravais Lattice (BL)
- All atoms are of the same kind
- All lattice points are equivalent

Non-Bravais Lattice (non-BL)
- Atoms can be of different kind
- Some lattice points are not equivalent
- A combination of two or more BL
Point

- 7 crystal systems
- 14 Bravais lattices
- 230 non-Bravais lattices
Thank you