

# Radiation from the LTB black hole

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Does a dynamical black hole embedded in a cosmological FRW background emit Hawking radiation where a globally defined event horizon does not exist? What are the differences to the Schwarzschild black hole? What about the first law of black hole mechanics? We face these questions using the LTB cosmological black hole model recently published. Using the Hamilton-Jacobi and radial null geodesic- methods suitable for dynamical cases, we show that it is the apparent horizon which contributes to the Hawking radiation and not the event horizon. The Hawking temperature is calculated using the two different methods giving the same result. The first law of LTB black hole dynamics and the thermal character of the radiation is also dealt with.

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## I. INTRODUCTION

Any black hole in the real universe is necessarily a dynamical one, i.e. it is neither stationary nor asymptotically flat. Therefore, its horizon has to be defined locally. The need to understand such dynamical and cosmological black holes has led to a revival of discussions on the concepts of black hole itself, its singularity, horizon, and thermodynamics [1]. Indeed, the conventional definition of black holes implies an asymptotically flat space-time and a global definition of the event horizon. The universe, however, is not asymptotically flat and a global definition of the horizon is not possible. The need for local definition of black holes and their horizons has led to concepts such as Hayward's trapping horizon [2], and Ashtekar and Krishnan's dynamical horizon [3]. It is not a trivial fact that in a specific example of a dynamical black hole any of these horizons may occur. In addition, we do not know if and how the Hawking radiation and the laws of black hole thermodynamics will apply to dynamical and cosmological black holes. Hence, it is of special interest to find out specific examples of dynamical black holes as a test bed for horizon problems.

Within a research program, we have already found an exact solution of Einstein equations based on the LTB solution [4] representing a dynamical black hole which is asymptotically FRW [7]. Different horizons and local mass definitions applied to this cosmological black hole is also reported [8, 9]. In this paper we are looking into the question of the Hawking radiation [10] and the first law of black hole dynamics in such a dynamical LTB black hole .

A first attempt to look into Hawking radiation from a 'cosmological black hole' is reported in [11]. There the authors consider the Einstein-Straus solution and the Sultana-Dyer one as cosmological black holes. The Einstein-Straus solution, however, is constructed such that it represents a 'frozen-out' Schwarzschild black hole within a FRW universe; it can not represent a dynamical black hole. The Sultana-Dyer solution is reproduced by the Schwarzschild metric through a conformal factor; it represent a FRW universe with a fixed black hole of a certain mass within it. Again, it can not reflect the features we expect from a dynamical black hole with the mass in-fall within an asymptotically FRW universe. Therefore, it is still an open question how the Hawking radiation and black hole dynamical laws will look like for a dynamical black hole within a FRW universe. We will look into this question in the case of the LTB cosmological black hole we have found as an exact solution of Einstein equation with mass in-fall, and without using a cut-and-paste technology of manifolds [7].

Now, Hawking's approach [10] based on quantum field theory, and applied to quasi cosmological black holes[11] is not a suitable method to calculate Hawking temperature in the case of proper dynamical black holes where one has to solve the field equations in a dynamical background. In such cases, like

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the LTB black hole [7], one should look for alternative approaches allowing to calculate the temperature of the Hawking radiation and the surface gravity. The so-called Hamilton-Jacobi (H-J) approach [13] has initiated methods suitable in such cases. The method, however, suffers from not being manifestly covariant. Using Kodama's formalism [21] for spherically symmetric space-times, and based on the Hamilton-Jacobi method, Hayward [20] has formulated a covariant form of the H-J approach and studied the quantum instability of a dynamical black hole [16, 17].

Another useful method is the semi-classical tunneling approach to Hawking radiation due to Parikh and Wilczek (PW) [12, 13, 19] using radial null geodesics. We will apply this radial null geodesic method, formulated originally for the static case, to our dynamic cosmological black hole and compare the result with that of the Hamilton-Jacobi method. The method, written in a gauge-invariant form, has led first to a Hawking temperature twice as large as the correct one [14]. It has, however, been shown that the correct Hawking temperature is regained by taking into account a contribution from the time coordinate upon crossing the horizon [15].

Among different definitions for the surface gravity of evolving horizons proposed in the past, the one formulated by Hayward [20] based on the Kodama's formalism [21] is the most suitable one to be used in our dynamic case. This leads us to the first law of black holes compatible with the Hawking temperature calculated by two different methods mentioned above.

The question of thermal character of the Hawking radiation will also be discussed within this formalism. As Parikh and Wilczek have already pointed out, the Hawking radiation is non-thermal when the energy conservation is enforced [12, 25]. As a result it has been shown by Zhang et al [26] that the total entropy becomes conserved and the black hole evaporation process is unitary.

We will introduce in section II the LTB black hole. In Section III, the covariant Hamilton-Jacobi tunneling method is introduced and applied to the LTB black hole. Section IV is devoted to the radial null geodesic method and its application to the LTB black hole. In Section V, the first law for the LTB black hole is derived. The thermal character of radiation is considered in section VI. We then conclude in section VII.

## II. INTRODUCING THE LTB BLACK HOLE

The cosmological LTB black hole is defined by a cosmological spherical symmetric isotropic solution having an overdense mass distribution within it [7]. The overdense mass distribution collapses due to the dynamics of the model leading to a black hole at the center of the structure. The LTB metric is the simplest spherically symmetric solution of Einstein equations representing an inhomogeneous dust distribution [4]. It may be written in synchronous coordinates as

$$ds^2 = -dt^2 + \frac{R'^2}{1+f(r)}dr^2 + R(t,r)^2 d\Omega^2, \quad (1)$$

representing a pressure-less perfect fluid satisfying

$$\rho(r,t) = \frac{2M'(r)}{R^2 R'}, \quad \dot{R}^2 = f + \frac{2M}{R}. \quad (2)$$

Here dot and prime denote partial derivatives with respect to the parameters  $t$  and  $r$  respectively. The angular distance  $R$ , depending on the value of  $f$ , is given by

$$\begin{aligned} R &= -\frac{M}{f}(1 - \cos\eta(r,t)), \\ \eta - \sin\eta &= \frac{(-f)^{3/2}}{M}(t - t_b(r)), \end{aligned} \quad (3)$$

$$\dot{R} = (-f)^{1/2} \frac{\sin(\eta)}{1 - \cos\eta}, \quad (4)$$

for  $f < 0$ , and

$$R = \left(\frac{9}{2}M\right)^{1/3}(t - t_b)^{2/3}, \quad (5)$$

for  $f = 0$ , and

$$\begin{aligned} R &= \frac{M}{f}(\cosh \eta(r, t) - 1), \\ \sinh \eta - \eta &= \frac{f^{3/2}}{M}(t - t_b(r)), \end{aligned} \quad (6)$$

for  $f > 0$ .

The metric is covariant under the rescaling  $r \rightarrow \tilde{r}(r)$ . Therefore, one can fix one of the three free functions of the metric, i.e.  $t_b(r)$ ,  $f(r)$ , or  $M(r)$ . The function  $M(r)$  corresponds to the Misner-Sharp mass in general relativity [8]. The  $r$  dependence of the bang time  $t_b(r)$  corresponds to a non-simultaneous big bang- or big-crunch-singularity.

There are two generic singularities of this metric, where the Kretschmann- and Ricci- scalar become infinite: the shell focusing singularity at  $R(t, r) = 0$ , and the shell crossing one at  $R'(t, r) = 0$ . However, there may occur that in the case of  $R(t, r) = 0$  the density  $\rho = \frac{M'}{R^2 R'}$  and the term  $\frac{M}{R^3}$  remain finite. In this case the Kretschmann scalar remains finite and there is no shell focusing singularity. Similarly, in the case of vanishing  $R'$  the term  $\frac{M'}{R'}$  may remain finite leading to a finite density and no shell crossing singularity either. Note that an expanding universe means generally  $\dot{R} > 0$ . However, in a region around the center it may happen that  $\dot{R} < 0$ , corresponding to the collapsing region.

The LTB metric may also be written in a form similar to the Painlevé form of the Schwarzschild metric. By taking the physical radius as a new coordinate using the relation  $dR = R' dr + \dot{R} dt$  one obtains

$$\begin{aligned} ds^2 &= \left(\frac{\dot{R}^2}{1+f} - 1\right)dt^2 + \frac{dR^2}{1+f} - \frac{2\dot{R}}{1+f}dRdt \\ &\quad + R(t, r)^2 d\Omega^2. \end{aligned} \quad (7)$$

The (t,R) coordinates are usually called the physical coordinates. In the case of  $f = 0$ , the metric is quite similar to the Painlevé metric.

### III. HAMILTON-JACOBI METHOD

The Hamilton-Jacobi method to calculate the Hawking radiation uses the fact that within the WKB approximation the tunneling probability for the classically forbidden trajectory from inside to outside the horizon is given by

$$\Gamma \propto \exp\left(-\frac{2}{\hbar}\text{Im } S\right), \quad (8)$$

where  $S$  is the classical action of the (massless) particle to the leading order in  $\hbar$  [13]. Note that the exponent has to be a scalar invariant, otherwise no physical meaning could be given to  $\Gamma$ . If, in particular, it has the form of a thermal emission spectrum with  $2\text{Im } S = \beta\omega$ , then both the inverse temperature  $\beta$  and the particle's energy  $\omega$  have to be scalars; otherwise no invariant meaning could be given to the horizon temperature, which would then not be an observable.

Now, let us use the Kodama formalism to introduce the necessary invariant quantities[16]. Any spherically symmetric metric can be expressed in the form

$$ds^2 = \gamma_{ij}(x^i)dx^i dx^j + R^2(x^i)d\Omega^2, \quad i, j \in \{0, 1\}, \quad (9)$$

where the two-dimensional metric

$$d\gamma^2 = \gamma_{ij}(x^i)dx^i dx^j \quad (10)$$

is referred to as the normal metric,  $x^i$  are associated coordinates, and  $R(x^i)$  is the areal radius considered as a scalar field in the normal two-dimensional space. Another relevant scalar quantity on this normal space is

$$\chi(x) = \gamma^{ij}(x)\partial_i R \partial_j R. \quad (11)$$

The dynamical trapping horizon,  $H$ , may be defined by

$$\chi(x) \Big|_H = 0, \quad \partial_i \chi \Big|_H \neq 0. \quad (12)$$

The Misner-Sharp gravitational mass is then given by

$$M(x) = \frac{1}{2}R(x)(1 - \chi(x)) , \quad (13)$$

which is an invariant quantity on the normal space. In the special case of the LTB metric this reduces to the Misner-Sharp mass  $M(r)$ . Note also that on the horizon  $M|_H = m = R_H/2$ . Now, there is always possible is such a spherically symmetric space time to define a preferred observer and its related invariant energy corresponding to the classical action of a particle. The Kodama vector [21], representing a preferred observer corresponding to the Killing vector in the static case, for the case of the LTB metric (9) is given by

$$K^i(x) = \frac{1}{\sqrt{-\gamma}}\varepsilon^{ij}\partial_j R, \quad K^\theta = 0 = K^\varphi . \quad (14)$$

Using this Kodama vector, we may introduce the invariant energy associated with a particle by means of its classical action being a scalar quantity on the normal space

$$\omega = K^i\partial_i S. \quad (15)$$

Note that during the process of horizon tunneling  $\omega$  is invariant independent of coordinates and is regular across the horizon. In the case of the eikonal approximation for massless wave field (geometric optics limits), which plays an important role in calculating the Hawking radiation using tunneling method [24], the classical action  $S$  for the massless particle satisfies the Hamilton-Jacobi equation

$$\gamma^{ij}\partial_i S\partial_j S = 0. \quad (16)$$

The relevant imaginary part of the classical action along the  $\gamma$  null curve is calculated in [16], where it has been shown that the tunneling rate (8) is valid for the future trapped horizon, and

$$\text{Im } S = \text{Im} \left( \int_\gamma dx^i \partial_i S \right) = \frac{\pi\omega_H}{\kappa_H}, \quad (17)$$

where  $\omega_H$  is the Kodama energy and  $\kappa_H$  is the dynamical surface gravity associated with the dynamical horizon:

$$\kappa_H = \frac{1}{2}\square_\gamma R|_H = \frac{1}{2\sqrt{-\gamma}}\partial_i(\sqrt{-\gamma}\gamma^{ij}\partial_j R)|_H. \quad (18)$$

These are scalar quantities in the normal space. Therefore, the leading term of the tunneling rate is invariant, as it should be for an observable quantity. The particle production rate then takes the thermal form  $\Gamma \sim e^{-\frac{\omega}{T}}$  with

$$T = \frac{\hbar\kappa_H}{2\pi}. \quad (19)$$

### A. Application to the LTB back hole

Assume the cosmological LTB black hole which has an infinite redshift surface satisfying the eikonal approximation condition for the Hamilton-Jacobi equation (16). It has been shown in [9] that the apparent horizon of this LTB black hole in its last stages is a slowly evolving horizon with the least mass in-fall due to the expanding background preventing the mass in-fall to the central black hole.

Now, from equations (1) and (2), and the definition of the surface gravity (18), we obtain

$$\kappa_H = \frac{m}{R^2} - \frac{m'}{2RR'} = \frac{1}{2R} - \frac{m'}{2RR'}, \quad (20)$$

where  $m$  is the Misner-Sharp mass on the horizon. Using this expression for the surface gravity, we obtain the Hawking temperature according to the Hamilton-Jacobi tunneling approach:

$$\begin{aligned} T &= \frac{\hbar\kappa_H}{2\pi} = \frac{\hbar}{4\pi} \frac{\sqrt{1+f}}{R'} \left[ -\frac{\partial_t(\dot{R}R')}{\sqrt{1+f}} + \partial_r(\sqrt{1+f}) \right] \\ &= \frac{\hbar}{4\pi} \left( \frac{f'}{2R'} - \ddot{R} - \frac{\dot{R}\dot{R}'}{R'} \right) = \frac{\hbar}{4\pi} \left( \frac{1}{R} - \frac{m'}{2RR'} \right). \end{aligned} \quad (21)$$

To relate this result to the temperature seen by the Kodama observer, we calculate first the Kodama vector (14). It is given by  $K^i = \frac{\sqrt{1+f}}{R'}(R', -\dot{R})$ . The equation  $|K| = \sqrt{-K_i K^i} = \sqrt{1+f - \dot{R}^2} = \sqrt{1 - \frac{2m}{R}}$  shows that the Kodama vector is a null vector on the horizon. The corresponding velocity vector is then given by  $\hat{K}^i = \frac{K^i}{|K|}$ . We then obtain the frequency measured by such an observer as  $\hat{\omega} = \hat{K}^i \partial_i S$ . The emission rate will take the thermal form

$$\Gamma \propto e^{-\frac{\hat{\omega}}{\hat{T}}} \quad (22)$$

which defines the temperature  $\hat{T}$ . It is then easily seen that the temperature for this observer is

$$\hat{T} = \frac{T}{\sqrt{1 - \frac{2m}{R}}}, \quad (23)$$

which diverges at the horizon. The invariant redshift factor  $\frac{1}{\sqrt{1 - \frac{2m}{R}}}$  is the same factor which appear in the light frequency on the horizon showing an infinite redshift to the observer in the infinity. Then  $T$  itself, being finite at the horizon, may be interpreted as the redshift-renormalized temperature [16].

#### IV. RADIAL NULL GEODESIC APPROACH

There is another approach to the Hawking radiation as a quantum tunneling process using WKB approximation for radial null geodesics tunneling out from near the horizon [12]. The imaginary part of the action is defined by

$$\begin{aligned} \text{Im}S &= \text{Im} \int_{r_{\text{in}}}^{r_{\text{out}}} p_r dr = \text{Im} \int_{r_{\text{in}}}^{r_{\text{out}}} \int_0^{p_r} dp'_r dr \\ &= \text{Im} \int_{r_{\text{in}}}^{r_{\text{out}}} \int_0^H \frac{-dH'}{\dot{r}} dr, \end{aligned} \quad (24)$$

using the Hamilton equation  $\dot{r} = \frac{dH}{dp_r}|_r$  with  $H$  being the Hamiltonian of the particle, i.e. the generator of the cosmic time  $t$ . Now, taking the tunneling probability as  $\Gamma \sim e^{-\frac{2}{\hbar} \text{Im}S}$ , being proportional to the Boltzmann factor  $e^{-\frac{\omega}{T}}$ , we find the Hawking temperature as

$$T_H = \frac{\omega \hbar}{2 \text{Im}S}. \quad (25)$$

It is easy to show that for a Schwarzschild black hole one obtain the correct expression  $T_H = \frac{\hbar}{8\pi M}$ .

It has been pointed out in [14], that  $2 \text{Im} \int_{r_{\text{in}}}^{r_{\text{out}}} p_r dr$  is not canonically invariant and thus it does not represent a proper observable. The object which is canonically invariant is  $\text{Im} \oint p_r dr$ , where the closed path goes across the horizon and back. Using this invariant definition and  $\Gamma \sim e^{-\frac{1}{\hbar} \text{Im} \oint p_r dr}$ , the Hawking temperature is found to be twice the original temperature. This discrepancy in the temperature has been resolved by considering temporal contribution to the tunneling amplitude. In the case of the Schwarzschild black hole, the temporal contribution to the action is found by changing Schwarzschild coordinates into Kruskal-Szekeres coordinates and then matching different Schwarzschild time coordinates across the horizon [15].

##### A. Application to the LTB black hole

We use the LTB space-time in physical coordinates. In this case the outgoing and ingoing null geodesics are given by

$$\frac{dR}{dt} = (\dot{R} \pm \sqrt{1+f}), \quad (26)$$

where the plus sign refers to the outgoing null geodesics. Now, expanding the above equation around the horizon, we obtain

$$\begin{aligned} \frac{dR}{dt} &= \sqrt{1+f} - \sqrt{1+f} \sqrt{1 - \frac{R-2m}{R(1+f)}} \\ &\cong \frac{R-2m}{2R \sqrt{1+f}}, \end{aligned} \quad (27)$$

where  $R - 2m$  is assumed to be a non-zero small quantity. Using the results for the radial null geodesics, we obtain for the imaginary part of the action corresponding to the LTB metric

$$\begin{aligned} \text{Im}S &= \text{Im} \int_{R_{\text{in}}}^{R_{\text{out}}} p_R dR \\ &= \text{Im} \int_{R_{\text{in}}}^{R_{\text{out}}} \int_0^H \frac{-dH'}{\dot{R}} dR \\ &= \text{Im} \int_{R_{\text{in}}}^{R_{\text{out}}} \int_0^H \frac{2R\sqrt{1+f}(-dH')}{R-2m} dR, \end{aligned} \quad (28)$$

where we have used the above expansion for  $\frac{dR}{dt}$  up to the first order of  $R - R_H = R - 2m$ . The corresponding Kodama invariant energy of the particle  $\omega = K^i S_i = \sqrt{1+f} \partial_t S$  is then calculated to be  $\omega = (\sqrt{1+f})dH'$ . We then use the expansion of  $R - 2m(r)$  in the form

$$\begin{aligned} R - 2m(r) &= \\ (1 - 2\frac{dm}{dR})(R - R|_H) - 2\frac{dm(r)}{dt}(t - t|_H) &= \\ ((1 - 2\frac{dm}{dR}) - \frac{2}{\frac{dR}{dt}|_{\text{null}}}\frac{dm(r)}{dt})(R - R|_H). \end{aligned} \quad (29)$$

Changing the variables  $(t, R)$  to  $(t, r)$ , makes it easier to use the expression for the surface gravity in synchronous coordinates. Putting  $\frac{dm}{dR} = \frac{m'}{R'}$ ,  $\frac{dm(r)}{dt} = m' \frac{\partial r}{\partial t}|_{R=\text{cont}}$ ,  $\frac{\partial r}{\partial t}|_{R=\text{cont}} = -\frac{\dot{R}}{R'}$ , and  $\frac{dR}{dt} = 2\dot{R}$  in (29) and (28), we obtain for the imaginary part of the action

$$\text{Im}S = \frac{w\pi}{(\frac{1}{2R} - \frac{m'}{2RR'})} = \frac{w\pi}{\kappa_H}, \quad (30)$$

leading to the Hawking temperature (20):

$$T = \frac{\hbar\kappa_H}{2\pi}. \quad (31)$$

This Hawking radiation temperature is the same as the one calculated in the last section using Hamilton-Jacobi approach [17]. The definition of the surface gravity used here has been essential to arrive at this result indicating it to be more useful than the other definitions in the literature [18].

Note that according to (2) and (7), the term  $R - 2m$  and accordingly the metric factor of  $dt^2$ , i.e.  $\frac{\dot{R}^2}{1+f} - 1$ , vanishes on the horizon, leading to  $\frac{dR}{dt} = 2\dot{R}$ . A fact which has not to be assumed while calculating the radial null geodesics, as has been done in [17]. Otherwise it would result to the vanishing of the imaginary part of the action and therefore no tunneling.

## V. FIRST LAW OF THE DYNAMICAL LTB BLACK HOLES

Using the tunneling approach for the Hawking radiation, we formulate now a first law of the LTB black hole. Consider the following invariant quantity in the normal space [33]

$$T^{(2)} = \gamma^{ij} T_{ij}, \quad (32)$$

where  $T_{ij}$  is the normal part of energy momentum tensor. Now, using the invariant surface gravity term in LTB given by (20), it is easy to show that on the dynamical horizon of our LTB black hole we have

$$\kappa_H = \frac{1}{2R_H} + 8\pi R_H T_H^{(2)}, \quad (33)$$

where we have used  $T_{00} = -\rho$  according to the section (3.1) and Einstein equations (2). The horizon area, the areal volume associated with the horizon, and their respective differentials are then given by

$$\mathcal{A}_H = 4\pi R_H^2, \quad d\mathcal{A}_H = 8\pi R_H dR_H, \quad (34)$$

$$V_H = \frac{4}{3}\pi R_H^3, \quad dV_H = 4\pi R_H^2 dR_H. \quad (35)$$

Substitution from above leads to

$$\frac{\kappa_H}{8\pi} d\mathcal{A}_H = d\left(\frac{R_H}{2}\right) + T_H^{(2)} dV_H. \quad (36)$$

Introducing the Misner-Sharp energy at the horizon, i.e.  $m = R_H/2$ , this can be recast in the form of a first law:

$$dm = \frac{\kappa_H}{2\pi} d\left(\frac{\mathcal{A}_H}{4}\right) - T_H^{(2)} dV_H = \frac{\kappa_H}{2\pi} ds_H - T_H^{(2)} dV_H, \quad (37)$$

where  $s_H = \mathcal{A}_H/4\hbar$  generalizes the Bekenstein-Hawking black hole entropy.

To see how this black hole first law is related to the Hawking radiation, we concentrate on two conserved currents which can be introduced in for LTB black hole. The first one is due to the Kodama vector  $K^a$ , and the corresponding conserved charge given by the area volume  $V = \int_{\sigma} K^a d\sigma_a = 4\pi R^3/3$ , where  $d\sigma_a$  is the volume form times a future directed unit normal vector of the space-like hypersurface  $\sigma_a$ . The second one may be defined by the energy-momentum density  $j^a = T_b^a K^b$  along the Kodama vector, and its corresponding conserved charge  $E = -\int_{\sigma} j^a d\sigma_a$  being equal to the Misner-Sharp energy. The total energy inside the apparent horizon can then be written as  $E_H = R|_H/2$ , which is the Misner-Sharp energy at the radius  $R = R|_H$  of our LTB black hole. The energy outside the region can be expressed as  $E_{>H} = -\int_{\sigma} T_b^a K^b d\sigma_a$ , where the integration extends from the apparent horizon to infinity. We may therefore express the total energy of the spacetime as

$$E_t = R|_H/2 - \int_{\sigma} T_b^a K^b d\sigma_a. \quad (38)$$

Consider now a tunneling process. The initial state before the tunneling defined by  $R = R|_H$  having an energy  $E_t^i$ , and the final one after the tunneling defined by  $R = R|_H + \delta R|_H$  having the energy  $E_t^f$ . According to the energy conservation, the Kodama energy change between the final and initial states of the tunneling process is then calculated to be

$$w = dE_t = E_t^f - E_t^i = \frac{\delta R|_H}{2} - \rho dV. \quad (39)$$

Substituting from (30) and (36) in the above equation, we obtain the tunneling rate  $\Gamma \sim e^{\frac{-2ImS}{\hbar}} = e^{\frac{-1}{4\hbar} \int_{s_i}^{s_f} d\mathcal{A}_H} = e^{\Delta s}$ , with  $\Delta s = s_f - s_i$  being the entropy change. Our discussion shows that the tunneling rate arises as a natural consequence of the unified first law of thermodynamics  $dE_H = T ds - T_H^{(2)} dV_H$  at the apparent horizon.

## VI. NON-THERMAL RADIATION FROM THE LTB BLACK HOLE

The question of how the formulas for black hole radiation are modified due to the self-gravitation of the radiation is dealt with in Kraus and Wilczek [25]. There it is shown that the Hawking radiation is non-thermal when the energy conservation is enforced. The particle in the particle hole system is treated as a spherical shell to have a workable model with the least degrees of freedom. The radiating black hole of mass  $M$  will be modeled as a shell with the energy  $\omega$  around the hole having the energy  $M(r) - \omega$ . We are going to adapt this model to our LTB black hole. Note first that due to the fact that our LTB black hole model is asymptotically Friedman, the ADM mass is not available to be fixed there. We therefore turn to the quasi-local Misner-Sharp mass. Let us then calculate the tunneling amplitude using this modification:

$$\begin{aligned} \text{Im}S &= \text{Im} \int_{R_{\text{in}}}^{R_{\text{out}}} p_R dR \\ &= \text{Im} \int_{R_{\text{in}}}^{R_{\text{out}}} \int_0^H \frac{-dH'}{\dot{R}} dR \\ &= \text{Im} \int_{2M(r)}^{2(M(r)-\omega)} dR \int_0^H \frac{-dH'}{\dot{R}}. \end{aligned} \quad (40)$$

Carrying out the first integral, we obtain

$$\begin{aligned} \text{Im}S &= \int_0^H \frac{\pi dH'}{\left(\frac{1}{4M-4H'} - \frac{m'}{(4M-4H')R'}\right)} \simeq \\ &= \frac{\pi H(1 - \frac{H}{2M})}{\left(\frac{1}{4M} - \frac{m'}{4MR'}\right)} = \frac{\pi\omega(1 - \frac{\omega}{2M})}{\kappa_H}, \end{aligned} \quad (41)$$

where we have used the fact  $\frac{\omega}{2M} \ll 1$  in the last step to expand the integrand and carry out the integral. The result shows the non-thermal character of the radiation.

It has been shown in [9] that the boundary of the LTB black hole becomes a slowly evolving horizon for  $R' \gg 1$ , with the surface gravity being equal to  $\kappa_H = \frac{1}{4M}$ , and an infinite redshift for the light coming out of this horizon. Therefore, using the above equation, the tunneling probability has the same form as in the case of Schwarzschild, i.e.  $\Gamma \sim \exp[-8\pi\omega(M - \frac{\omega}{2})]$ . Having this form of the tunneling probability we may refer to Zhang et al. [26] who have shown that this form of the non-thermal radiation leads to the conservation of the total entropy.

## VII. CONCLUSIONS

Within a research program to understand more in detail the LTB cosmological black hole, its similarities and differences to the Schwarzschild black hole, we have calculated the Hawking radiation from this dynamical black hole by using the tunneling methods suitable for dynamical cases. It turns out that for the LTB black hole the Hamilton-Jacobi and the radial null geodesic-method both lead to the same tunneling rate. It turns out that it is not the event horizon but the future outer trapping horizon that contributes to this Hawking radiation.

Formulation of a first law for the LTB black hole and the tunneling amplitude show that the tunneling rate has a direct relation to the change of the LTB black hole entropy. Assuming the energy conservation for the LTB black hole's slowly evolving horizon, we show that the radiation is non-thermal and that the entropy is conserved during the radiation.

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